Between Contact and Support: Introducing a logic for image schemas and directed movement

Maria M. Hedblom[1,2](✉), Oliver Kutz[1], Till Mossakowski[2], and Fabian Neuhaus[2]

[1] Free University of Bozen-Bolzano, Italy
mhedblom@unibz.it
[2] Otto-von-Guericke University Magdeburg, Germany

Abstract. Cognitive linguistics introduced image schemas as a missing link between embodied experiences and high-level conceptualisation in language and metaphorical thinking. They are described as the abstract spatio-temporal relationships that function as conceptual building blocks for everyday concepts and events. Although there is increasing interest in the area of cognitively motivated artificial intelligence, where image schemas are suggested to be a core piece in the puzzle to model human-level conceptualisation and reasoning, so far rather few formal logical approaches can be found in the literature, in particular regarding attention to the dynamic aspects of image schemas. A fundamental problem here is that the typical mainstream approaches in contemporary KR do not map well to various scenarios found in image schema modelling. In this paper, we introduce a spatio-temporal logic for 'directed movement of objects', with the aim to model formally image schematic events such as Blockage, Caused_Movement and 'bouncing'.

1 Introduction

Embodied cognition states that all cognition occurs as a consequence of the body's sensorimotor experiences with its environment [24]. Within this framework the theory of image schemas was introduced as a link between embodied experiences and mental representations [11,16]. As natural language understanding remains one of the major obstacles in the advancement of artificial intelligence, there has been an increased interest in utilising image schemas as a stepping stone towards simulating human cognition through formal representations.

Image schemas may be described as spatio-temporal relationships between objects and their environment [15]. In developmental psychology, they are thought to develop as infants are repeatedly exposed to certain spatial relationships [20]. In cognitive linguistics, image schemas are primarily studied as conceptual skeletons that underlie metaphors, analogical reasoning and abstract concepts [8].

However, research on image schemas raises many challenges. As image schemas are abstract mental patterns, there exists currently no complete and agreed upon list of image schemas. Despite this lack of common ground in the research field, some commonly investigated image schemas are CONTAINMENT, SUPPORT and SOURCE_PATH_GOAL.[3]

Regarding identification, classification and formalisation of image schemas, there are three main considerations to be taken into account. The first is that image schemas are rarely clear-cut notions in themselves, but rather appear as networks of closely associated relationships [10]. The second, related issue, is that while image schemas are, by definition, the most generic conceptual building blocks, they also function as building blocks for each other. For example, when investigating established image schemas such as BLOCKAGE or CAUSED_MOVEMENT, these more complicated image schemas can be dissected into simpler image schemas such as SOURCE_PATH_GOAL and CONTACT. A third, unfortunately often neglected, aspect of image schemas is their dynamic nature. Not only are they spatially complex, but they are also temporally complex, and they involve force dynamics.

In this paper we try to address these issues by applying methods from *qualitative spatial reasoning* (QSR) [19]. QSR is an area of AI that studies spatio-temporal reasoning that approximates human common-sense understanding of space. Research in QSR is, typically, about a given set of spatial relations (e.g., Left, Right, FrontOf, Behind, Above, Below), their logical dependencies, and how they may be used to describe complex spatial arrangements.

The main hypothesis of this paper is the following: image schemas may be represented in a language that combines features from several existing QSR theories. This representation enables the analysis of dependencies and connections between image schemas and it enables us to take into account the temporal dimension of image schemas. Therefore, this paper introduces a novel logic, called ISL^M, for image schemas combining the Region Connection Calculus (RCC-8), Qualitative Trajectory Calculus (QTC), Cardinal Directions (CD) and Linear Temporal Logic (LTL).

To illustrate the modelling capabilities of ISL^M, we introduce a static Two-Object image schema family involving CONTACT and SUPPORT. Considering the addition of movement and temporal change, we then show how ISL^M can express more complex and dynamic image-schematic scenarios: BLOCKAGE, the ceasing of MOVEMENT_OF_OBJECT; CAUSED_MOVEMENT, the beginning of MOVEMENT_OF_OBJECT through an impact with another object; and 'bouncing', the event in which an object encountering BLOCKAGE reverses in the opposite trajectory.

2 A Logic for Directed Movement ISL^M

In general, the rich models of time investigated in more cognitively-driven studies on how humans understand time in poetry, everyday cognition, language in general, and communication can not be mapped easily to existing temporal logic approaches [3,5]. The limitations of off-the-shelf calculi also extends to the

[3] We write established image schemas in small caps.

spatial domain, and to standard spatio-temporal combined logics, see e.g. [13]. The well known Region Connection Calculus (RCC) has been used extensively in qualitative spatial reasoning [4]. However, cognitive studies have supported the claim that humans do not typically make, or accept, some of the distinctions inherent in the RCC calculus [12]. A simpler calculus (usually called RCC-5), can be obtained by removing the distinction between e.g., 'proper part' and 'tangential proper part', however collapsing the logic to pure mereology [17]. At the other end of the spectrum is the work of [1], who attempted to model the image schema of containment from the linguistic perspective. To map the pertinent distinctions made in natural language concerning variations of containment (such as 'surround', 'enclose'), they needed to *extend axiomatically* the RCC theory to capture the identified eight different kinds of containers.

2.1 The spatial dimension – topology of regions

Before we can move on to the modelling scenarios sketched in the introduction, we need to introduce the logical framework in some detail. First, following the work that has been laid out by amongst others [7,1], the Region Connection Calculus (RCC) is used as a method to represent basic topological spatial relationships. Here we are using the RCC-8 relation [21]. The reason is that a mere mereology would not suffice for modelling image schemas as we need a means to express that two objects touch each other (EC).[4]

2.2 The spatial dimension – cardinal directions

Directions can be absolute or relative. Usually, left and right denote relative directions [23], which however are conceptually and computationally much more complicated than (absolute) cardinal directions [18] like North or West. We here assume a naive egocentric view (i.e. with a fixed observer that is not part of the model), from which directions like left/right, front/behind and above/below can be recognised as cardinal directions. This leads to six binary predicates on objects: *Left*, *Right*, *FrontOf*, *Behind*, *Above* and *Below*. Note that these relations are unions of base relations in a three-dimensional cardinal direction calculus as in [18], and the latter can be recovered from these relations by taking suitable intersections and complements (for example, it is possible that none of the above six relations hold, which happens to be the case if two regions are equal or largely overlap).

2.3 The movement dimension

In order to take the dynamic aspects of the image schemas into account, the Qualitative Trajectory Calculus (QTC) [26] is used to represent how two objects

[4] For this paper, we only use EC and DC (disconnected). However, when looking at image schemas such as CONTAINMENT additional members of these qualitative relations are needed. Moreover, proximity spaces and point-free Whiteheadian systems based on 'connection' [25] will be considered as alternatives in future work.

relate in terms of movement. In its variant QTC_{B1D}, the trajectories of objects are described in relation to one another. While [26] use nine different relations[5], these are composed of two independent parts, with three possibilities for each part. We here simplify the calculus by only considering these three possibilities: if object O_1 moves towards O_2's position, this is represented as $O_1 \rightsquigarrow O_2$, if O_1 moves away from O_2's position, this is represented as $O_1 \hookleftarrow O_2$, while O_1 being at rest with respect to O_2's position is expressed as $O_1 \mid\!\circ\, O_2$. This way of writing the relative movement of two objects is intuitive and expressive. The calculus of [26] can be recovered by taking intersections of these relations, combining the description of the movement of O_1 with respect to O_2's position with the description of the movement of O_2 with respect to O_1's position. For example, $O_1 \rightsquigarrow O_2 \wedge O_2 \hookleftarrow O_1$ is denoted as O_1-$+O_2$ in [26].

With QTC, we can speak about relative movement for a given time point. What is missing is the ability to speak about the future.

2.4 The temporal dimension

We use the simple linear temporal logic LTL [14,22], but interpreted over the reals instead of over the naturals. The syntax is as follows:

$$\varphi \ ::= \ p \mid \top \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi U \varphi$$

$\varphi\, U\, \psi$ reads as "φ holds, until ψ holds". As is standard in temporal logic, we can define the following derived operators:

- $F\varphi$ (at some time in the future, φ) is defined as $\top U \varphi$,
- $G\varphi$ (at all times in the future, φ) is defined as $\neg F \neg \varphi$.

We moreover use $\rightarrow$ for material implication, $\leftrightarrow$ for biimplication, and $\veebar$ for exclusive disjunction.

2.5 The combined logic ISL^M

Syntax of ISL^M: The syntax of ISL^M is defined over the combined languages of RCC-8, QTC_{B1D}, cardinal direction (CD), and linear temporal logic (LTL) over the reals, with 3D Euclidan space assumed for the spatial domain. Note that we need LTL over real-time in order to interpret QTC relations, the semantics of which assume continuous time. ISL^M therefore stands for '**I**mage **S**chema **L**ogic' and $M = \langle \text{RCC-8}, QTC_{B1D}, CD, LTL, \text{3D-Euclid}\rangle$. The combination of the spatial and temporal modalities follows the temporalisation strategy of [6].

Signatures (vocabularies) A signature $\Sigma = (R_r, R_f)$ consist of a set R_r of rigid and a set R_f of flexible object names. In the context of modelling image schemas, though not playing a central role in the present paper, this will be useful to handle the modelling of objects that do not change their position nor

---

[5] The reason for using nine relations is the wish to obtain a partition of the space of all relations between two objects, as is usually done in qualitative spatial reasoning.

their extension during a period of time (like a house) vs. objects that essentially have to change (like a moving ball or a balloon being inflated).

Σ**-Sentences** are LTL temporal formulas (see Sect. 2.4) built over (ground) atomic formulas taken from the union of RCC-8 statements (see Sect. 2.1), 3D cardinal directions (see Sect. 2.2) and QTC_{B1D} (see Sect. 2.3), plus the forces predicate. Atomic formulas apply predicates to object names $O_1, O_2, \ldots \in R_r \cup R_f$.[6]

Example 1. Here are a few examples of well-formed sentences that can be written in this language (and might be considered true in specific scenarios). Note, however, that none of them are valid (i.e. true in all models), but can be valid in scenarios where the geometry of objects and possible movements are further restricted in the description of the semantics, or can alternatively be used to prescribe admissible models.

– $\mathit{FrontOf}(a,b) \wedge \mathbf{F}\neg\mathit{FrontOf}(a,b) \longrightarrow \mathbf{F}(a \rightsquigarrow b \vee a \hookleftarrow b \vee b \rightsquigarrow a \vee b \hookleftarrow a)$ 'If a is in front of b, but ceases to do so in the future, then sometime in the future, either a or b must move with respect to the other object's original position';
– $\mathit{Above}(a,b) \wedge \mathbf{G}a \mid\!\circ\; b \longrightarrow \mathbf{G}\mathit{Above}(a,b)$ 'If a is above b and never moves relative to b, it will be always above b'. Note that this sentence is not valid: consider e.g. that a circles around b with constant distance. However, it holds if for example a and b always stay on the same line (that is, their relative movement is 1D only);
– $DC(a,b) \wedge \mathbf{G}a \hookleftarrow b \longrightarrow \mathbf{G}DC(a,b)$ 'If a is disconnected to b and always moves away from it, it will always stay disconnected to b'. This is actually a validity.

Semantics of ISL^M: We interpret the combined logic ISL^M spatially over regions in $\mathbb{R}^3$ and temporally over the real line. Note that we need continuous time in order to interpret QTC properly.[7] An interpretation (model) $M = (\_|_M\_, \mathit{forces}_M)$ consists of

– a function
$$\_|_M\_ : (R_r \cup R_f) \times \mathbb{R} \to \mathcal{P}(\mathbb{R}^3)$$
such that $r|_M t$ is the region covered by object r at time t, and $r|_M t$ does not depend on t for $r \in R_r$, and
– a relation
$$\mathit{forces}_M(t) \subseteq (R_r \cup R_f) \times (R_r \cup R_f)$$
such that $\mathit{forces}_M(t)(r,s)$ if object r imposes force on object s at time t.

Given a formula φ and a time point $t \in \mathbb{R}$, we define its satisfaction $M,t \models \varphi$ as follows. If φ is an atomic formula, we define

[6] Introducing variables and (controlled, cognitively-motivated) quantification over objects is left for a future extension of the logic.
[7] Studying alternatives to this choice is part of future work.

– If R is an RCC-8 relation, $M, t \models R(r, s)$ holds if $r|_M t$ is in relation R with $s|_M t$, following the RCC-8 semantics in [21].
– if R is a cardinal direction relation, then
  - $M, t \models \mathit{Left}(r, s)$ holds if $\inf\{x \mid (x, y, z) \in r|_M t\} \geq \sup\{x \mid (x, y, z) \in s|_M t\}$.
    $M, t \models \mathit{Right}(r, s)$ holds if $M, t \models \mathit{Left}(s, r)$ holds.
  - $M, t \models \mathit{FrontOf}(r, s)$ holds if $\inf\{y \mid (x, y, z) \in r|_M t\} \geq \sup\{y \mid (x, y, z) \in s|_M t\}$.
    $M, t \models \mathit{Behind}(r, s)$ holds if $M, t \models \mathit{FrontOf}(s, r)$ holds.
  - $M, t \models \mathit{Above}(r, s)$ holds if $\inf\{z \mid (x, y, z) \in r|_M t\} \geq \sup\{z \mid (x, y, z) \in s|_M t\}$.
    $M, t \models \mathit{Below}(r, s)$ holds if $M, t \models \mathit{Above}(s, r)$ holds.
– $M, t \models \mathsf{forces}(r, s)$ holds if $\mathit{forces}_M(t)(r, s)$.
– QTC_{B1D} formulas are interpreted as in [26], but over regions as moving objects. Therefore, we define distance between regions as follows, based on the usual Euclidean distance d:

$$d(Y, Z) = \inf\{d(y, z) \mid y \in Y, z \in Z\}$$

  Then, given region names r and s, exactly one of three cases occurs:
  - $M, t \models r \rightsquigarrow s$ iff r is moving towards s's position, that is, if
    $\exists t_1(t_1 < t \wedge \forall t^-(t_1 < t^- < t \rightarrow d(r|_M t^-, s|_M t) > d(r|_M t, s|_M t)))\wedge$
    $\exists t_2(t < t_2 \wedge \forall t^+(t < t^+ < t_2 \rightarrow d(r|_M t, s|_M t) > d(r|_M t^+, s|_M t)))$
  - $M, t \models r \hookleftarrow s$ iff r is moving away from s's position, that is, if
    $\exists t_1(t_1 < t \wedge \forall t^-(t_1 < t^- < t \rightarrow d(r|_M t^-, s|_M t) < d(r|_M t, s|_M t)))\wedge$
    $\exists t_2(t < t_2 \wedge \forall t^+(t < t^+ < t_2 \rightarrow d(r|_M t, s|_M t) < r(k|_M t^+, s|_M t)))$
  - $M, t \models r \mid\circ\, s$ iff r is of stable distance with respect to s, that is, in all other cases. Note that stable distance does not imply absence of relative movement. For example, consider that r moves around s but keeps the distance stable (e.g. a satellite moves around the earth). It could even be that r is inside s and moves there (and the distance is constantly 0).

Satisfaction of complex formulas is inherited from LTL:

– for atomic p, $M, t \models p$ has been defined above
– $M, t \models \neg\varphi$ iff not $M, t \models \varphi$
– $M, t \models \varphi \wedge \psi$ iff $M, t \models \varphi$ and $M, t \models \psi$
– $M, t \models \varphi U \psi$ iff for some $u > t$, $M, u \models \psi$ and $M, v \models \varphi$ for all $v \in [t, u)$.

Finally, φ holds in M, denoted $M \models \varphi$, if for all $t \in \mathbb{R}$, $M, t \models \varphi$.

A notable feature of this semantics is that the timepoint where relative movement starts or stops does itself not belong to the set of timepoints where relative movement occurs. As a consequence, relative movement implies disconnectedness, that is

$$(r \rightsquigarrow s \vee r \hookleftarrow s) \rightarrow DC(r, s)$$

is a validity. If two objects are e.g. externally connected (EC), their distance is 0, and therefore, they cannot move away from or towards to each other. Suppose that the distance of two objects is 0 at time 1, and relative movement starts at time 1. Then the two objects will be EC and stable (unmoved) at time 1, but will be DC and in relative movement for the interval $(1, 1 + \varepsilon]$ for some $\varepsilon > 0$.

3 Verticality, Attraction and the static ‘Two-Object’ family

Before moving on to image schemas that encompass two objects in spatial contact with each other, here referred to as a subset of the ‘Two-Object family’, we need to introduce two other important image schemas: the Verticality schema and the Attraction schema. This is important as we need image schematic components from these to successfully build the Two-Object family.

Verticality is believed to be one of the earliest image schemas to be learned based on the human body’s vertical axis and the perceived effect gravity has on objects [11]. In its static form, Verticality represents orientation and relational notions of above and below.

Likewise, image schemas such as Attraction and conceptual structures that encompass physical forces are experienced and conceptualised in the first six months [20]. Objects fall to the ground, not because of Verticality in itself, but because of the “Attraction objects have to the ground”[8]. Attraction is part of the force group of image schemas [11], and while it is more complicated than simple ‘force towards/from’, we ascertain that for the purpose of extracting conceptual primitives of force relations, Attraction provides a good starting point.

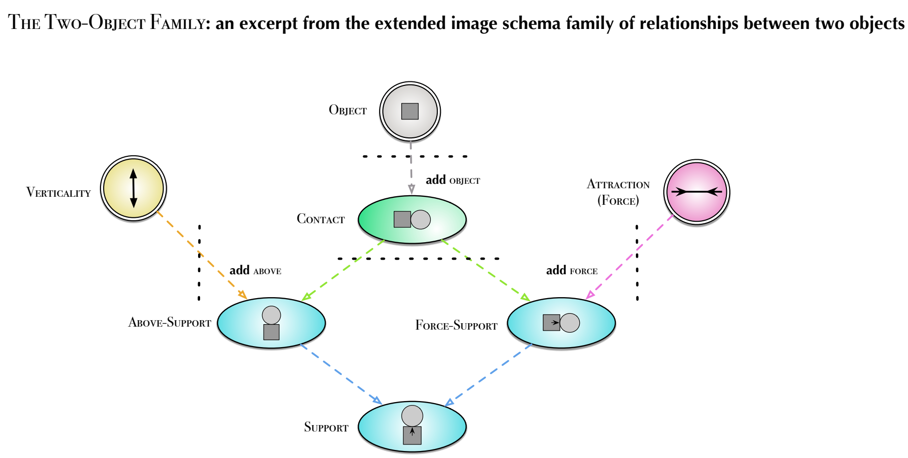


Fig. 1: How Contact can be hierarchically connected to Support through addition of spatial primitives from Verticality and Attraction.

Figure 1 illustrates how some of the image schemas involving two objects can be formally developed. In the most general form, Contact represents the object

[8] Children naturally do not understand gravity, yet they learn to predict that objects are ‘forced’ downwards.

relation in which two objects are physically touching. This is strongly related to the notion of SUPPORT. However, it is unlikely that infants understand the forces acting in an image schema like SUPPORT. Therefore, in many scenarios it might be sufficient to speak of SUPPORT in terms of CONTACT with 'above' orientation. By merging the image schema CONTACT with the static form of VERTICALITY ('above') we get an Above-SUPPORT image schema.

If instead of VERTICALITY, force is added to CONTACT, another weaker SUPPORT can be distinguished, Force-SUPPORT. Here the important aspect is that the supporting object offers physical support, which does not have to be vertical. For instance, a plank that 'leans against a wall' also captures a form of SUPPORT. The most specific and traditional form of SUPPORT is constructed when both Above-SUPPORT and Force-SUPPORT are combined.[9]

In the next section we will demonstrate how the CONTACT and SUPPORT notions as presented in the Two-Object family can be formally represented.

3.1 Formalising the static image schemas CONTACT and SUPPORT

As discussed above, CONTACT is the most general image schema in which two objects have a (physical) connection to each other. For CONTACT, the object relationship is without any force dynamics neither does it contain any topological or orientational requirements.

CONTACT is one of the simplest image schemas to conceptualise and consequently also to formally represent using our logic. For our purposes, CONTACT is formalised as two regions, here represented by object names O_1 and O_2, touching, which is represented in RCC-8 as $\text{CONTACT}(O_1, O_2) \leftrightarrow EC(O_1, O_2)$.

SUPPORT requires a more involved formalisation given that ATTRACTION or 'force' and/or VERTICALITY and 'above'-ness are involved to keep one object in contact with another object. Therefore, we first need a formal representation of both 'above' and 'force'. VERTICALITY in terms of above (and below) orientation is expressed with the following predicate $Above(x, y)$ where x is above y, and $forces(x, y)$ demonstrate how x puts physical force on y (see Section 2.5 for details). Given this, we can formalise the two weaker SUPPORT versions, Above-SUPPORT and Force-SUPPORT, and when these are merged the union correspond to universal and more complete version of SUPPORT (see Figure 1).

$$Above\text{-}\textsc{Support}(O_1, O_2) \leftrightarrow EC(O_1, O_2) \wedge Above(O_1, O_2)$$
$$Force\text{-}\textsc{Support}(O_1, O_2) \leftrightarrow EC(O_1, O_2) \wedge \mathsf{forces}(O_1, O_2)$$
$$\textsc{Support}(O_1, O_2) \leftrightarrow EC(O_1, O_2) \wedge Above(O_1, O_2) \wedge \mathsf{forces}(O_1, O_2)$$

In the next section we proceed to use these formalisations to demonstrate some more intricate examples by formalising the dynamic image schematic events BLOCKAGE, CAUSED_MOVEMENT and bouncing.

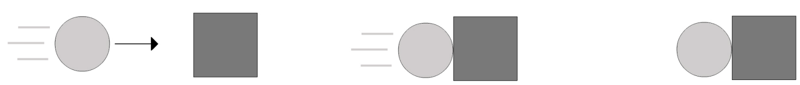
[9] The authors acknowledge that additional CONTACT and SUPPORT relationships may exist that have not been considered in this paper.

4 The dynamic image schemas Blockage, Caused_Movement and bouncing

Before we move on to dynamic image schema combinations, we need to introduce the image schemas for movement. The Source_Path_Goal schema can be dissected into a range of different simpler (and more complex) forms of movements (see [9] for an overview). In our logic we simplify movement by using the QTC primitives.

4.1 Formalisations of the narratives behind Blockage, Caused_Movement and bouncing

Following the arguments presented in [2] where complex image schemas and simple events emerge as consequences of combinations of simpler image schemas, we now proceed to demonstrate how using the logic presented above yields a formal rendering of Blockage, Caused_Movement and 'bouncing'.



(a) O_1 On Path Toward O_2 (b) O_1 BLOCKed by O_2 (c) O_1 in Contact with O_2

Fig. 2: Illustrations of the three time intervals of Blockage.

Blockage. The simplest form of blocked movement is the scenario in which the movement of an object simply ceases to exist. While Blockage is considered an image schema in its own right, it is also possible to describe blockage using a series of simple image-schematic events: Movement_Of_Object[10], Contact and forcefollowed by the lack of Movement_Of_Object, see Figure 2. Formalised, it reads:

$$\text{On\_Path\_Toward}(O_1, O_2) = (O_1 \rightsquigarrow O_2 \wedge DC(O_1, O_2)) \qquad \text{((a) } O_1 \text{ on Path toward } O_2\text{)}$$

$$\text{BLOCKed\_By}(O_1, O_2) = (O_1 \mid\!\circ\, O_2 \wedge O_2 \mid\!\circ\, O_1 \wedge \textit{Force}\text{-Support}(O_1, O_2)) \qquad \text{((b) } O_1 \text{ BLOCKed by } O_2\text{)}$$

$$\text{In\_Contact}(O_1, O_2) = (O_1 \mid\!\circ\, O_2 \wedge O_2 \mid\!\circ\, O_1 \wedge EC(O_2, O_1)) \qquad \text{((c) } O_1 \text{ in Contact with } O_2\text{)}$$

The temporal scenario of 'blocked movement' is temporally captured as follows:

$$\text{On\_Path\_Toward}(O_1, O_2) \wedge \mathbf{F}\big(\,\text{BLOCKed\_By}(O_1, O_2) \;\wedge \mathbf{G}(\text{In\_Contact}(O_1, O_2))\,\big)$$

[10] Alternatively, it is possible to make it more specific by determining also the path that the object is moving on, namely through Movement_Along_Path.

Here, the nested future time operator guarantees that these events happen in the correct temporal order.

As these first steps until contact happens between two objects reoccur for all the subsequent scenarios, we will make repeated use of these defined predicates of On_PATH_Toward(O_1, O_2) and BLOCKed_By(O_1, O_2). One interesting thing to note here is that formalised and in combination with motion, BLOCKAGE works much like Force-SUPPORT. Compare the definition for SUPPORT and the definition for BLOCKed_By. The only difference is the addition of a temporal aspect through the lack of movement. This is an interesting observation, as our experience is affected by the physical world, meaning that gravitational pull could be viewed as a sort of 'downward' movement and that all SUPPORT is simply BLOCKAGE of that movement.

CAUSED_MOVEMENT. There are more scenarios that can result from BLOCKAGE than the static relation of CONTACT between the moving object and the blocking object, presented above. One of the first more 'complex' image schemas that appear in the literature is CAUSED_MOVEMENT. Namely the spatio-temporal relationship that results from one object colliding with another and causing that object to move.

Simplified, the image schema comes in three different forms. First, in the scenario in which the hitting object comes to rest while the hit object continues onward (e.g. as in a well executed billiards chock) referred to as "Pure_CM". Second, in which both objects continue in disjoint forward movement, "Pursuit_CM". There is also a third scenario in which the objects continue forward movement together, "Joint_CM". However, as CAUSED_MOVEMENT focus on the second object, this third form is currently ignored as it implies 'pushing' and agency of the first object, a modality not yet present in the logic.

Formalised, it reads:

$$\text{On\_PATH\_Toward}(O_1, O_2) \qquad (O_1 \text{ on PATH toward } O_2)$$
$$\text{BLOCKed\_By}(O_1, O_2) \qquad (O_1 \text{ BLOCKed by } O_2)$$

CAUSED_MOVEMENT alternative ending one, Pure_CM(see Figure 3a) followed by alternative ending two Pursuit_CMin which both objects move forward (see Figure 3b):

$$\text{Pure\_CM}(O_1, O_2) = O_2 \hookleftarrow O_1 \wedge O_1 \mid\!\circ\, O_2 \wedge DC(O_1, O_2)$$
$$\text{Pursuit\_CM}(O_1, O_2) = O_1 \rightsquigarrow O_2 \wedge O_2 \hookleftarrow O_1 \wedge DC(O_1, O_2)$$
$$((a)\ O_2 \text{ moves away from } O_1,\ O_1 \text{ is at rest in respect of } O_2)$$
$$((b)\ O_1 \text{ moves towards } O_2 \text{ which moves away from } O_1)$$

In temporal representation, the full scenario of CAUSED_MOVEMENT looks as follows:

$$\text{On\_PATH\_Toward}(O_1, O_2) \wedge$$
$$\mathbf{F}\big(\text{ BLOCKed\_By}(O_1, O_2) \ \wedge \mathbf{F}(\text{Pure\_CM}(O_1, O_2) \ \veebar \ \text{Joint\_CM}(O_1, O_2)\ \big)$$

(a) O_2 move away from O_1 (b) O_1, O_2 move forward

Fig. 3: Illustrations of the two formalised alternative endings of CAUSED_MOVEMENT.

Bouncing Another natural scenario that happens as one object hits another, is 'bouncing'. In comparison to CAUSED_MOVEMENT the object of interest here is not the object that is hit but rather the object that is doing the hitting.

The formalisations below correspond to the picture in Figure 4a.

$$\text{On\_PATH\_Toward}(O_1, O_2) \qquad \text{((a) } O_1 \text{ on PATH toward } O_2\text{)}$$
$$\text{BLOCKED\_By}(O_1, O_2) \qquad \text{((b) } O_1 \text{ BLOCKED by } O_2\text{)}$$
$$\text{Bouncing}(O_1, O_2) = O_1 \hookleftarrow O_2 \wedge O_2 \mid\!\circ\, O_1 \wedge DC(O_1, O_2)$$
$$\text{((c) } O_1 \text{ on PATH from } O_2 \text{ which is at rest in respect of } O_1\text{)}$$

In full temporal representation the scenario looks as follows:

$$\text{On\_PATH\_Toward}(O_1, O_2) \wedge \mathbf{F}\big(\ \text{BLOCKED\_By}(O_1, O_2) \wedge \mathbf{F}(\mathit{Bouncing}(O_1, O_2))\ \big)$$

Combination of CAUSED_MOVEMENT and bouncing Another, quite natural scenario is the combination of *bouncing* with CAUSED_MOVEMENT. In this scenario the hitting object O_1 bounces on O_2 while at the same time the impact pushes the blocking object away(see Figure 4b). Formalised, it reads:

$$\text{On\_PATH\_Toward}(O_1, O_2) \qquad \text{((a) } O_1 \text{ on PATH toward } O_2\text{)}$$
$$\text{BLOCKED\_By}(O_1, O_2) \qquad \text{((b) } O_1 \text{ BLOCKED by } O_2\text{)}$$
$$\text{Bouncing\_CM}(O_1, O_2) = O_1 \hookleftarrow O_2 \wedge O_2 \hookleftarrow O_1 \wedge DC(O_1, O_2)$$
$$\text{((c) } O_1 \text{ and } O_2 \text{ are on PATHs away from each other)}$$

(a) O_1 bounces on O_2 (b) O_1 bounces and O_2 moves forward

Fig. 4: Illustrations of the results of bouncing respectively the result of the combination of CAUSED_MOVEMENT and bouncing.

5 Conclusion and future work

Developing a formal theory of image schemas is essential for several areas of cognitively motivated AI, such as computational conceptual blending [10], and can also be used, as done in this paper, to better analyse formally distinctions motivated by empirical research.

The paper has presented a novel way to formally represent image schemas in a language that combines features from several existing QSR theories. The representation illuminates the internal structure of image schemas. One result is that there are families of image schemas that contain closely related image schemas (e.g., the different kinds of SUPPORT). Further, some image schemas are part of others (e.g., Force-SUPPORT is part of BLOCKAGE). The formalism also allows us to represent the different stages of an image schema and, thus, represent its temporal dimension.

The formalisation builds on a combination of Linear Temporal Logic (LTL) [14], Qualitative Trajectory Calculus [26], Cardinal Directions, and the RCC-8 relations [21] that previously were used to formally approach image schemas. The combination approach following [6] allows for controlled interaction between the dimensions, and a decidability and complexity analysis is part of future work.

The modelling approach is illustrated with the 'Two Object' family, capturing some static relationships between two objects, as well as using it to narratively express the dynamic image schemas and simple events 'BLOCKAGE', 'CAUSED_MOVEMENT' and 'bouncing', originally introduced in [2].

Future work includes to use the logic to model concrete scenarios, and to illustrate how it supports common sense reasoning based on image schemas and e.g. the logical prediction of future events such as BLOCKAGE, as well as extend the logic with other modalities, for instance, the notion of agency.

References

1. Bennett, B., Cialone, C.: Corpus Guided Sense Cluster Analysis: a methodology for ontology development (with examples from the spatial domain). In: 8th Int. Conf. on Formal Ontology in Information Systems (FOIS). Frontiers in Artificial Intelligence and Applications, vol. 267, pp. 213–226. IOS Press (2014)
2. Besold, T.R., Hedblom, M.M., Kutz, O.: A narrative in three acts: Using combinations of image schemas to model events. BICA 19, 10–20 (2017)
3. Boroditsky, L.: Metaphoric structuring: Understanding time through spatial metaphors. Cognition 75(1), 1–28 (2000)
4. Cohn, A.G., Bennett, B., Gooday, J., Gotts, N.: RCC: a calculus for region based qualitative spatial reasoning. GeoInformatica 1, 275–316 (1997)
5. Coulson, S., Cánovas, C.P.: Understanding Timelines: Conceptual Metaphor and Conceptual Integration. Cognitive Semiotics 5(1-2), 198–219 (2014)
6. Finger, M., Gabbay, D.M.: Adding a Temporal Dimension to a Logic System. Journal of Logic, Language and Information 1, 203–233 (1993)
7. Galton, A.: The Formalities of Affordance. In: Bhatt, M., Guesgen, H., Hazarika, S. (eds.) Proc. of workshop Spatio-Temporal Dynamics. pp. 1–6 (2010)

8. Hampe, B., Grady, J.E.: From perception to meaning: Image schemas in cognitive linguistics, vol. 29. Walter de Gruyter, Berlin (2005)
9. Hedblom, M.M., Kutz, O., Neuhaus, F.: Choosing the Right Path: Image Schema Theory as a Foundation for Concept Invention. JAGI 6(1), 22–54 (2015)
10. Hedblom, M.M., Kutz, O., Neuhaus, F.: Image schemas in computational conceptual blending. Cognitive Systems Research 39, 42–57 (2016)
11. Johnson, M.: The Body in the Mind: The Bodily Basis of Meaning, Imagination, and Reason. The University of Chicago Press, Chicago and London (1987)
12. Knauff, M., Rauh, R., Renz, J.: A cognitive assessment of topological spatial relations: Results from an empirical investigation. In: Hirtle, S.C., Frank, A.U. (eds.) Spatial Information Theory: A Theoretical Basis for GIS, LNCS, vol. 1329, pp. 193–206. Springer (1997)
13. Kontchakov, R., Kurucz, A., Wolter, F., Zakharyaschev, M.: Spatial logic+ temporal logic=? In: Handbook of spatial logics, pp. 497–564. Springer (2007)
14. Kröger, F., Merz, S.: Temporal Logic and State Systems (Texts in Theoretical Computer Science. An EATCS Series). Springer (2008)
15. Kuhn, W.: An Image-Schematic Account of Spatial Categories. In: Winter, S., Duckham, M., Kulik, L., Kuipers, B. (eds.) Spatial Information Theory, LNCS, vol. 4736, pp. 152–168. Springer (2007)
16. Lakoff, G.: Women, Fire, and Dangerous Things. What Categories Reveal about the Mind. The University of Chicago Press (1987)
17. Lehmann, F., Cohn, A.G.: The EGG/YOLK reliability hierarchy: Semantic data integration using sorts with prototypes. In: Proc. Conf. on Information Knowledge Management. pp. 272–279. ACM Press (1994)
18. Ligozat, G.: Reasoning about cardinal directions. J. Vis. Lang. Comput. 9(1), 23–44 (1998)
19. Ligozat, G.: Qualitative Spatial and Temporal Reasoning. Wiley (2011)
20. Mandler, J.M.: The Foundations of Mind: Origins of Conceptual Thought: Origins of Conceptual Though. Oxford University Press, New York (2004)
21. Randell, D.A., Cui, Z., Cohn, A.G.: A spatial logic based on regions and connection. In: Proc. of the 3rd Int. Conf. on Knowledge Representation and Reasoning (KR-92) (1992)
22. Reynolds, M.: The complexity of temporal logic over the reals. Annals of Pure and Applied Logic 161(8), 1063–1096 (2010)
23. Scivos, A., Nebel, B.: The Finest of its Class: The Natural, Point-Based Ternary Calculus $\mathcal{LR}$ for Qualitative Spatial Reasoning. In: Spatial Cognition. pp. 283–303 (2004)
24. Shapiro, L.: Embodied Cognition. New problems of philosophy, Routledge, London and New York (2011)
25. Vakarelov, D., Düntsch, I., Bennett, B.: A note on proximity spaces and connection based mereology. In: Proceedings of the International Conference on Formal Ontology in Information Systems (FOIS). pp. 139–150 (2001)
26. Weghe, N.V.D., Cohn, A.G., Tré, G.D., Maeyer, P.D.: A qualitative trajectory calculus as a basis for representing moving objects in geographical information systems. Control and cybernetics 35(1), 97–119 (2006)